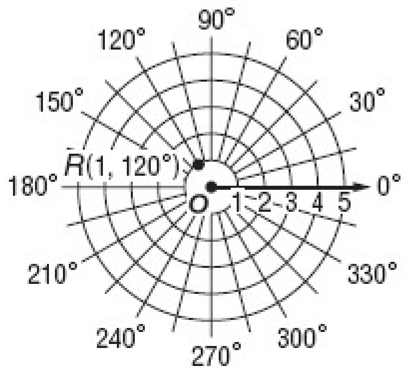
9-1 Polar Coordinates

Graph each point on a polar grid.

1. $R(1, 120°)$

SOLUTION:

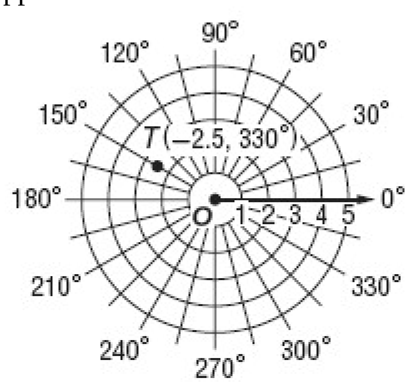
Because θ = 120°, locate the terminal side of a 120° angle with the polar axis as its initial side. Because r = 1, plot a point 1 unit from the pole along the terminal side of the angle.



2. $T(-2.5, 330°)$

SOLUTION:

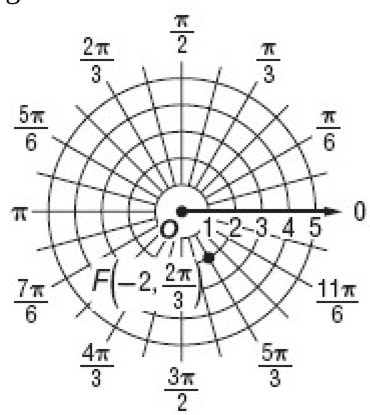
Because θ = 330°, locate the terminal side of a 330° angle with the polar axis as its initial side. Because r = −2.5, plot a point 2.5 units from the pole in the opposite direction of the terminal side of the angle.



3. $F\left(-2, \frac{2\pi}{3}\right)$

SOLUTION:

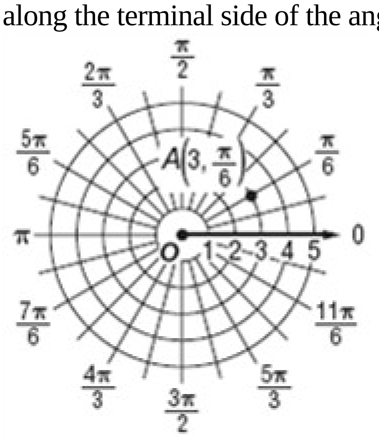
Because $\theta = \frac{2\pi}{3}$, locate the terminal side of a $\frac{2\pi}{3}$ angle with the polar axis as its initial side. Because r = −2, plot a point 2 units from the pole in the opposite direction of the terminal side of the angle.



4. $A\left(3, \frac{\pi}{6}\right)$

SOLUTION:

Because $\theta = \frac{\pi}{6}$, locate the terminal side of a $\frac{\pi}{6}$ angle with the polar axis as its initial side. Because r = 3, plot a point 3 units from the pole along the terminal side of the angle.



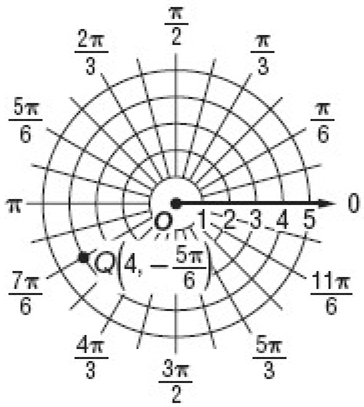
5. $Q\left(4, -\frac{5\pi}{6}\right)$

SOLUTION:

Because $\theta = -\frac{5\pi}{6}$, locate the terminal side of a $-\frac{5\pi}{6}$ angle with the polar axis as its initial side.

$$-\frac{5\pi}{6} + 2\pi = \frac{7\pi}{6}$$

Because $r = 4$, plot a point 4 units from the pole along the terminal side of the angle.



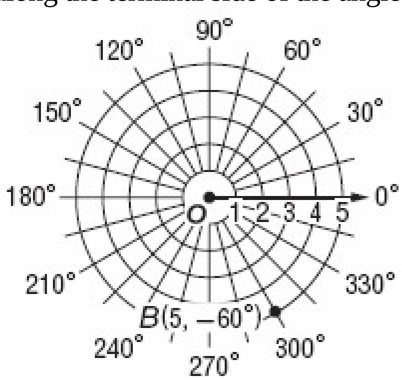
6. $B(5, -60°)$

SOLUTION:

Because $\theta = -60°$, locate the terminal side of a $-60°$ angle with the polar axis as its initial side.

$-60° + 360° = 300°$.

Because $r = 5$, plot a point 5 units from the pole along the terminal side of the angle.



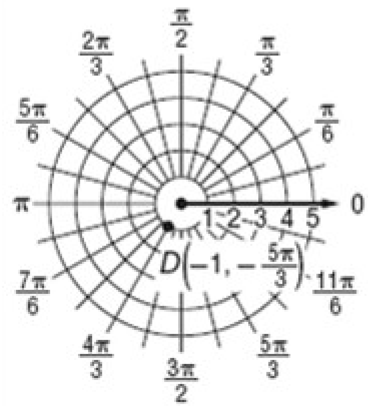
7. $D\left(-1, -\frac{5\pi}{3}\right)$

SOLUTION:

Because $\theta = -\frac{5\pi}{3}$, locate the terminal side of a $-\frac{5\pi}{3}$ angle with the polar axis as its initial side.

$$-\frac{5\pi}{3} + 2\pi = \frac{\pi}{3}$$

Because $r = -1$, plot a point 1 unit from the pole in the opposite direction of the terminal side of the angle.



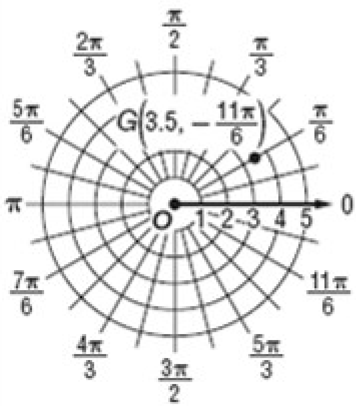
8. $G\left(3.5, -\frac{11\pi}{6}\right)$

SOLUTION:

Because $\theta = -\frac{11\pi}{6}$, locate the terminal side of a $-\frac{11\pi}{6}$ angle with the polar axis as its initial side.

$-\frac{11\pi}{6} + 2\pi = \frac{\pi}{6}$

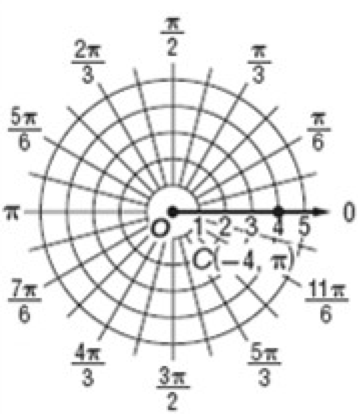
Because $r = 3.5$, plot a point 3.5 units from the pole along the terminal side of the angle.



9. $C(-4, \pi)$

SOLUTION:

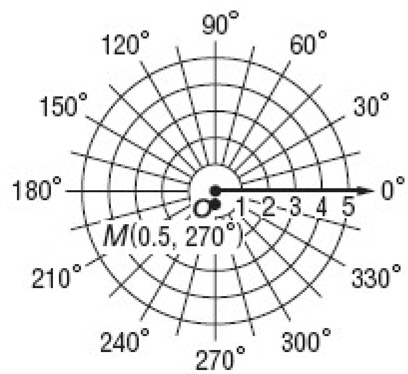
Because $\theta = \pi$, locate the terminal side of a π angle with the polar axis as its initial side. Because $r = -4$, plot a point 4 units from the pole in the opposite direction of the terminal side of the angle.



10. $M(0.5, 270°)$

SOLUTION:

Because $\theta = 270°$, locate the terminal side of a 270° angle with the polar axis as its initial side. Because $r = 0.5$, plot a point 0.5 unit from the pole along the terminal side of the angle.



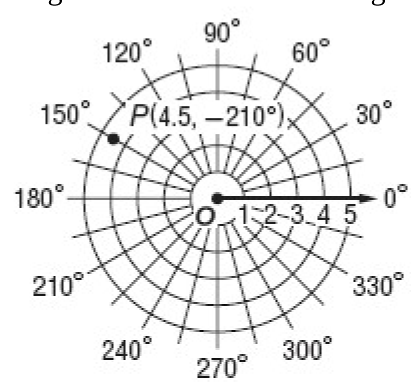
11. $P(4.5, -210°)$

SOLUTION:

Because $\theta = -210°$, locate the terminal side of a $-210°$ angle with the polar axis as its initial side.

$-210° + 360° = 150°$

Because $r = 4.5$, plot a point 4.5 units from the pole along the terminal side of the angle.



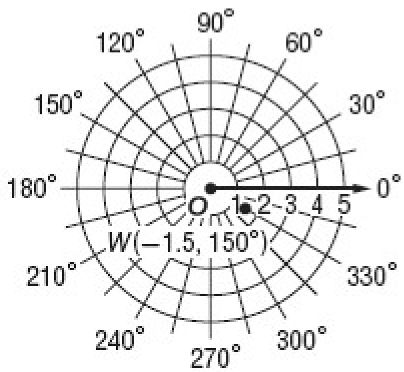
9-1 Polar Coordinates

12. $W(-1.5, 150°)$

SOLUTION:

Because θ = 150°, locate the terminal side of a 150° angle with the polar axis as its initial side.

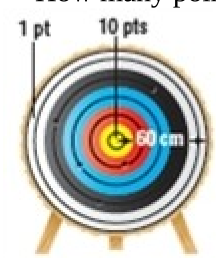
Because $r = -1.5$, plot a point 1.5 units from the pole in the opposite direction of the terminal side of the angle.



13. **ARCHERY** The target in competitive target archery consists of 10 evenly spaced concentric circles with score values from 1 to 10 points from the outer circle to the center. Suppose an archer using a target with a 60-centimeter radius shoots arrows at (57, 45º), (41, 315º), and (15, 240º).

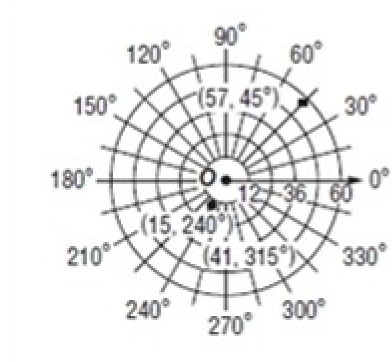
a. Plot the points where the archer's arrows hit the target on a polar grid.

b. How many points did the archer earn?



SOLUTION:

a. For (57, 45°), locate the terminal side of a 45°-angle with the polar axis as its initial side. Then plot a point 57 units from the pole along the terminal side of the angle. For (41, 315°), locate the terminal side of a 315°-angle with the polar axis as its initial side. Then plot a point 41 units from the pole along the terminal side of the angle. For (15, 240°), locate the terminal side of a 240°-angle with the polar axis as its initial side. Then plot a point 15 units from the pole along the terminal side of the angle.



b. If the 10 circles are concentric and evenly spaced, then distance from one circle to the next is 6 centimeters. Therefore, the point values for an arrow landing between certain distances are given below.

0 cm–6 cm	10 pts
6 cm–12 cm	9 pts
12 cm–18 cm	8 pts
18 cm–24 cm	7 pts
24 cm–30 cm	6 pts
30 cm–36 cm	5 pts
36 cm–42 cm	4 pts
42 cm–48 cm	3 pts
48 cm–54 cm	2 pts
54 cm–60 cm	1 pt

The shot at (57, 45°) earned 1 pt, at (41, 315°) earned 4 pts, and at (15, 240°) earned 8 pts for a total of 13 points.

Find three different pairs of polar coordinates that name the given point if $-360° \le \theta \le 360°$ or $-2\pi \le \theta \le 2\pi$.

14. (1, 150°)

SOLUTION:

For the point (1, 150°), the other three representations are as follows.

$$(1, 150°) = (1, 150° - 360°)$$
$$= (1, -210°)$$
$$(1, 150°) = (-1, 150° + 180°)$$
$$= (-1, 330°)$$
$$(1, 150°) = (-1, 150° - 180°)$$
$$= (-1, -30°)$$

9-1 Polar Coordinates

15. $(-2, 300°)$

SOLUTION:

For the point $(-2, 300°)$, the other three representations are as follows.

$$(-2, 300°) = (2, 300° - 180°)$$
$$= (2, 120°)$$
$$(-2, 300°) = (2, 300° - 540°)$$
$$= (2, -240°)$$
$$(-2, 300°) = (-2, 300° - 360°)$$
$$= (-2, -60°)$$

16. $\left(4, -\frac{7\pi}{6}\right)$

SOLUTION:

For the point $\left(4, -\frac{7\pi}{6}\right)$, the other three representations are as follows.

$$\left(4, -\frac{7\pi}{6}\right) = \left(4, -\frac{7\pi}{6} + 2\pi\right)$$
$$= \left(4, \frac{5\pi}{6}\right)$$
$$\left(4, -\frac{7\pi}{6}\right) = \left(-4, -\frac{7\pi}{6} + \pi\right)$$
$$= \left(-4, -\frac{\pi}{6}\right)$$
$$\left(4, -\frac{7\pi}{6}\right) = \left(-4, -\frac{7\pi}{6} + 3\pi\right)$$
$$= \left(-4, \frac{11\pi}{6}\right)$$

17. $\left(-3, \frac{2\pi}{3}\right)$

SOLUTION:

For the point $\left(-3, \frac{2\pi}{3}\right)$, the other three representations are as follows.

$$\left(-3, \frac{2\pi}{3}\right) = \left(3, \frac{2\pi}{3} + \pi\right)$$
$$= \left(3, \frac{5\pi}{3}\right)$$
$$\left(-3, \frac{2\pi}{3}\right) = \left(3, \frac{2\pi}{3} - \pi\right)$$
$$= \left(3, -\frac{\pi}{3}\right)$$
$$\left(-3, \frac{2\pi}{3}\right) = \left(-3, \frac{2\pi}{3} - 2\pi\right)$$
$$= \left(-3, -\frac{4\pi}{3}\right)$$

18. $\left(5, \frac{11\pi}{6}\right)$

SOLUTION:

For the point $\left(5, \frac{11\pi}{6}\right)$, the other three representations are as follows.

$$\left(5, \frac{11\pi}{6}\right) = \left(5, \frac{11\pi}{6} - 2\pi\right)$$
$$= \left(5, -\frac{\pi}{6}\right)$$
$$\left(5, \frac{11\pi}{6}\right) = \left(-5, \frac{11\pi}{6} - \pi\right)$$
$$= \left(-5, \frac{5\pi}{6}\right)$$
$$\left(5, \frac{11\pi}{6}\right) = \left(-5, \frac{11\pi}{6} - 3\pi\right)$$
$$= \left(-5, -\frac{7\pi}{6}\right)$$

9-1 Polar Coordinates

19. $\left(-5, -\frac{4\pi}{3}\right)$

SOLUTION:

For the point $\left(-5, -\frac{4\pi}{3}\right)$, the other three representations are as follows.

$$\left(-5, -\frac{4\pi}{3}\right) = \left(5, -\frac{4\pi}{3} + 3\pi\right)$$
$$= \left(5, \frac{5\pi}{3}\right)$$
$$\left(-5, -\frac{4\pi}{3}\right) = \left(5, -\frac{4\pi}{3} + \pi\right)$$
$$= \left(5, -\frac{\pi}{3}\right)$$
$$\left(-5, -\frac{4\pi}{3}\right) = \left(-5, -\frac{4\pi}{3} + 2\pi\right)$$
$$= \left(-5, \frac{2\pi}{3}\right)$$

20. $(2, -30°)$

SOLUTION:

For the point $(2, -30°)$, the other three representations are as follows.

$$(2, -30^\circ) = (2, -30^\circ + 360^\circ)$$
$$= (2, 330^\circ)$$
$$(2, -30^\circ) = (-2, -30^\circ + 180^\circ)$$
$$= (-2, 150^\circ)$$
$$(2, -30^\circ) = (-2, -30^\circ - 180^\circ)$$
$$= (-2, -210^\circ)$$

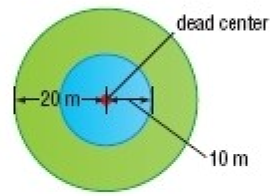
21. $(-1, -240°)$

SOLUTION:

For the point $(-1, -240°)$, the other three representations are as follows.

$$(-1, -240^\circ) = (1, -240^\circ - 180^\circ)$$
$$= (1, 300^\circ)$$
$$(-1, -240^\circ) = (1, -240^\circ + 180^\circ)$$
$$= (1, -60^\circ)$$
$$(-1, -240^\circ) = (-1, -240^\circ - 360^\circ)$$
$$= (-1, 120^\circ)$$

22. **SKYDIVING** In competitive accuracy landing, skydivers attempt to land as near as possible to "dead center," the center of a target marked by a disk 2 meters in diameter.

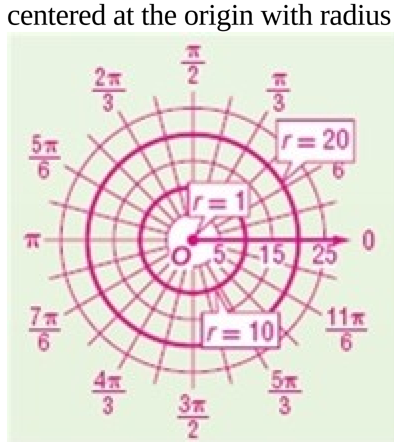


a. Write polar equations representing the three target boundaries.
b. Graph the equations on a polar grid.

SOLUTION:

a. In the polar coordinate system, a circle centered at the origin with a radius a units has equation $r = a$. Dead center has a radius of 1 meter. Its equation is therefore $r = 1$. The equations of the 10- and 20-radius circles are $r = 10$ and $r = 20$, respectively.

b. For $r = 1$, draw a circle centered at the origin with radius 1. For $r = 10$, draw a circle centered at the origin with radius 10. For $r = 20$, draw a circle centered at the origin with radius 20.

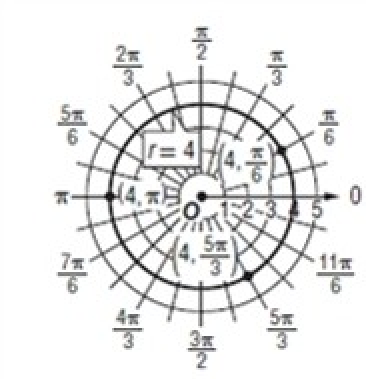


Graph each polar equation.

23. $r = 4$

SOLUTION:

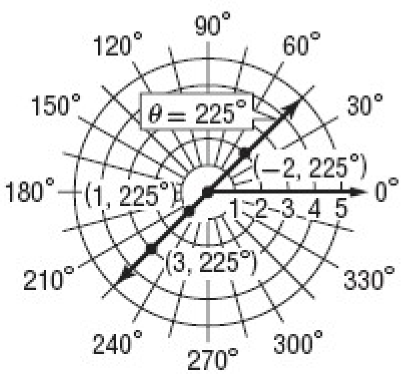
The solutions of $r = 4$ are ordered pairs of the form $(4, \theta)$, where θ is any real number. The graph consists of all points that are 4 units from the pole, so the graph is a circle centered at the origin with radius 4.



24. $\theta = 225°$

SOLUTION:

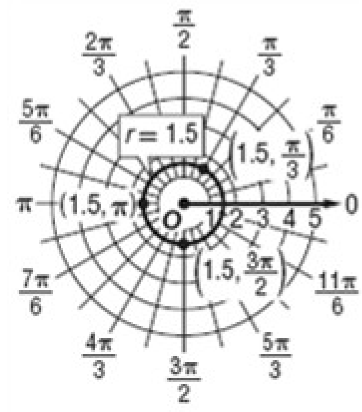
The solutions of $\theta = 225°$ are ordered pairs of the form $(r, 225°)$, where r is any real number. The graph consists of all points on the line that make an angle of 225° with the positive polar axis.



25. $r = 1.5$

SOLUTION:

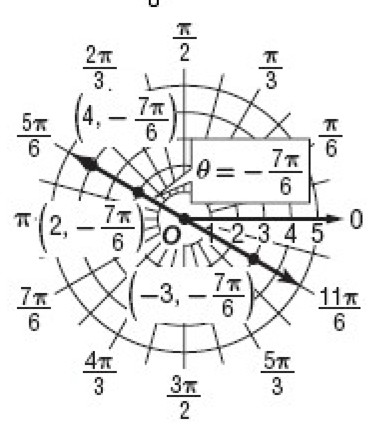
The solutions of $r = 1.5$ are ordered pairs of the form $(1.5, \theta)$, where θ is any real number. The graph consists of all points that are 1.5 units from the pole, so the graph is a circle centered at the origin with radius 1.5.



26. $\theta = -\frac{7\pi}{6}$

SOLUTION:

The solutions of $\theta = -\frac{7\pi}{6}$ are ordered pairs of the form $\left(r, -\frac{7\pi}{6}\right)$, where r is any real number. The graph consists of all points on the line that make an angle of $-\frac{7\pi}{6}$ with the positive polar axis.

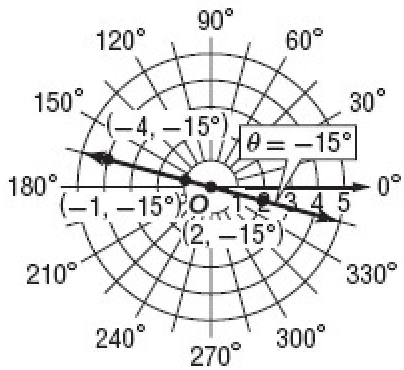


9-1 Polar Coordinates

27. $\theta = -15°$

SOLUTION:

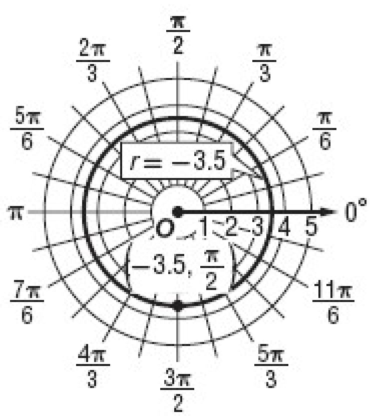
The solutions of $\theta = -15°$ are ordered pairs of the form $(r, -15°)$, where r is any real number. The graph consists of all points on the line that make an angle of $-15°$ with the positive polar axis.



28. $r = -3.5$

SOLUTION:

The solutions of $r = -3.5$ are ordered pairs of the form $(-3.5, \theta)$, where θ is any real number. The graph consists of all points that are 3.5 units from the pole, so the graph is a circle centered at the origin with radius 3.5.



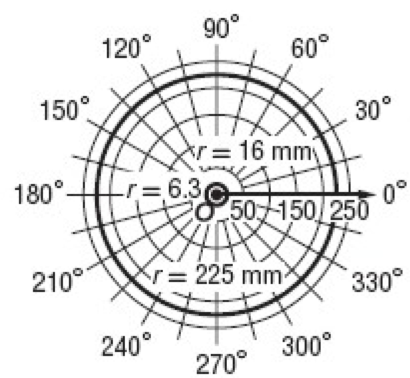
29. **DARTBOARD** A certain dartboard has a radius of 225 millimeters. The bull's-eye has two sections. The 50-point section has a radius of 6.3 millimeters. The 25-point section surrounds the 50-point section for an additional 9.7 millimeters.

a. Write and graph polar equations representing the boundaries of the dartboard and these sections.

b. What percentage of the dartboard's area does the bull's-eye comprise?

SOLUTION:

a. In the polar coordinate system, a circle centered at the origin with a radius a units has equation $r = a$. The dartboard has a radius of 225 mm, so its boundary equation is $r = 225$.
The 50-point section has a radius of 6.3 mm, so its boundary equation is $r = 6.3$. The 25-point section has a radius of 6.3 + 9.7 or 16 mm, so its boundary equation is $r = 16$. For $r = 225$, draw a circle centered at the origin with radius 225. For $r = 6.3$, draw a circle centered at the origin with radius 6.3. For $r = 16$, draw a circle centered at the origin with radius 16.



b. The bull's-eye has a radius of 16 mm. Find the ratio of the area of the bull's-eye to the area of the entire dartboard.

$$\begin{aligned}\frac{A_{\text{bull's-eye}}}{A_{\text{dartboard}}} &= \frac{\pi\left(r_{\text{bull's-eye}}\right)^2}{\pi\left(r_{\text{dartboard}}\right)^2} \\ &= \frac{\pi(16)^2}{\pi(225)^2} \\ &= \frac{256}{50625} \\ &\approx 0.005\end{aligned}$$

About 0.5%

9-1 Polar Coordinates

Find the distance between each pair of points.

30. (2, 30°), (5, 120°)

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{2^2 + 5^2 - 2(2)(5)\cos(120° - 30°)}$$
$$= \sqrt{4 + 25 - 20\cos(90°)}$$
$$\approx 5.39$$

31. $\left(3, \frac{\pi}{2}\right), \left(8, \frac{4\pi}{3}\right)$

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{3^2 + 8^2 - 2(3)(8)\cos\left(\frac{4\pi}{3} - \frac{\pi}{2}\right)}$$
$$= \sqrt{9 + 64 - 48\cos\left(\frac{5\pi}{6}\right)}$$
$$\approx 10.70$$

32. (6, 45°), (–3, 300°)

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{6^2 + (-3)^2 - 2(6)(-3)\cos(300 - 45)}$$
$$= \sqrt{36 + 9 + 36\cos(255)}$$
$$\approx 5.97$$

33. $\left(7, -\frac{\pi}{3}\right), \left(1, \frac{2\pi}{3}\right)$

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{7^2 + 1^2 - 2(7)(1)\cos\left[\frac{2\pi}{3} - \left(-\frac{\pi}{3}\right)\right]}$$
$$= \sqrt{49 + 1 - 14\cos(\pi)}$$
$$= 8$$

34. $\left(-5, \frac{7\pi}{6}\right), \left(4, \frac{\pi}{6}\right)$

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{(-5)^2 + 4^2 - 2(-5)(4)\cos\left(\frac{\pi}{6} - \frac{7\pi}{6}\right)}$$
$$= \sqrt{25 + 16 + 40\cos(-\pi)}$$
$$= 1$$

35. (4, –315°), (1, 60°)

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{4^2 + 1^2 - 2(4)(1)\cos[60 - (-315)]}$$
$$= \sqrt{16 + 1 - 8\cos 375}$$
$$\approx 3.05$$

36. (–2, –30°), (8, 210°)

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{(-2)^2 + 8^2 - 2(-2)(8)\cos[210 - (-30)]}$$
$$= \sqrt{4 + 64 + 32\cos(240)}$$
$$\approx 7.21$$

37. $\left(-3, \frac{11\pi}{6}\right), \left(-2, \frac{5\pi}{6}\right)$

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{(-3)^2 + (-2)^2 - 2(-3)(-2)\cos\left(\frac{5\pi}{6} - \frac{11\pi}{6}\right)}$$
$$= \sqrt{9 + 4 - 12\cos(-\pi)}$$
$$= 5$$

38. $\left(1, -\frac{\pi}{4}\right), \left(-5, \frac{7\pi}{6}\right)$

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{1^2 + (-5)^2 - 2(1)(-5)\cos\left[\frac{7\pi}{6} - \left(-\frac{\pi}{4}\right)\right]}$$
$$= \sqrt{1 + 25 + 10\cos\left(\frac{17\pi}{12}\right)}$$
$$\approx 4.84$$

39. (7, –90°), (–4, –330°)

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{7^2 + (-4)^2 - 2(7)(-4)\cos[-330 - (-90)]}$$
$$= \sqrt{49 + 16 + 56\cos(-240)}$$
$$\approx 6.08$$

40. $\left(8, -\frac{2\pi}{3}\right), \left(4, -\frac{3\pi}{4}\right)$

SOLUTION:

Use the Polar Distance Formula.

$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{8^2 + 4^2 - 2(8)(4)\cos\left[-\frac{3\pi}{4} - \left(-\frac{2\pi}{3}\right)\right]}$$
$$= \sqrt{64 + 16 - 64\cos\left(-\frac{\pi}{12}\right)}$$
$$\approx 4.26$$

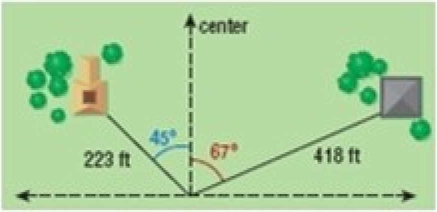
41. (–5, 135°), (–1, 240°)

SOLUTION:

Use the Polar Distance Formula.

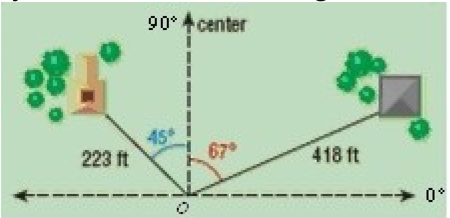
$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{(-5)^2 + (-1)^2 - 2(-5)(-1)\cos(240 - 135)}$$
$$= \sqrt{25 + 1 - 10\cos(105)}$$
$$\approx 5.35$$

42. **SURVEYING** A surveyor mapping out the land where a new housing development will be built identifies a landmark 223 feet away and 45° left of center. A second landmark is 418 feet away and 67º right of center. Determine the distance between the two landmarks.



SOLUTION:

Place the surveyor at the origin of a polar coordinate system as shown in the diagram below.



The first landmark makes a (90 – 67)° or 23°-angle with the polar axis and is located 418 ft along the angle's terminal side. The second landmark makes a (90 + 45)° or 135°-angle with the polar axis and is located 223 ft along the angle's terminal side. Therefore coordinates representing the first and second locations are (418, 23°) and (223, 135°), respectively. Use the polar distance formula to find the distance between the two landmarks.

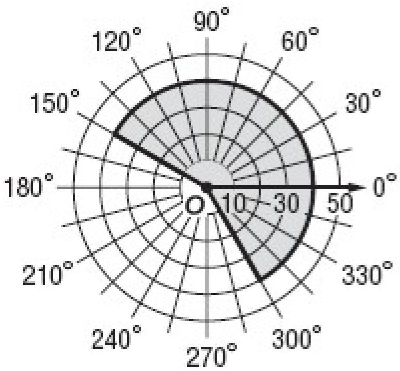
$$\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{418^2 + 223^2 - 2(418)(223)\cos(135° - 23°)}$$
$$\approx 542.5$$

The distance between the two landmarks is about 542.5 ft.

43. **SURVEILLANCE** A mounted surveillance camera oscillates and views part of a circular region determined by $-60° \le \theta \le 150°$ and $0 \le r \le 40$, where r is in meters.
a. Sketch a graph of the region the security camera can view on a polar grid.
b. Find the area of the region.

SOLUTION:

a. The parameters $-60° \le \theta \le 150°$ and $0 \le r \le 40$ describe a sector of a circle with radius 40 units from −60° to 150°.



b. The area of a sector of a circle with radius r and central angle θ is $A = \frac{1}{2}r^2\theta$, where θ is measured in radians.

$150° - (-60°) = 210°$

$210° \times \frac{\pi}{180°} = \frac{7\pi}{6}$ radians

Therefore, the area of the sector is $\frac{1}{2}(40)^2\left(\frac{7\pi}{6}\right) \approx$ 2932.2 m^2.

Find a different pair of polar coordinates for each point such that $0 \le \theta \le 180°$ or $0 \le \theta \le \pi$.

44. (5, 960°)

SOLUTION:

Let $P(r, \theta) = (5, 960°)$.

We need to subtract 960 by $180k$, such that the result is between 0 and 180. Test multiples of 180.

$$\begin{aligned} 960 - 5(180) &= 960 - 900 \\ &= 60 \end{aligned}$$

$k = 5$

Since k is odd, we need to replace r with $-r$ to obtain the correct polar coordinates.

$P(5, 960°) = (-5, 60°)$

45. $\left(-2.5, \frac{5\pi}{2}\right)$

SOLUTION:

Let $P(r, \theta) = \left(-2.5, \frac{5\pi}{2}\right)$.

We need to subtract $\frac{5\pi}{2}$ from πk, such that the result is between 0 and π. Test multiplies of π.

$$\begin{aligned} \frac{5\pi}{2} - 2\pi &= \frac{5\pi}{2} - \frac{4\pi}{2} \\ &= \frac{\pi}{2} \end{aligned}$$

Since k is even, use r to obtain the correct polar coordinates.

$P(r, \theta) = \left(-2.5, \frac{\pi}{2}\right)$

46. $\left(4, \frac{11\pi}{4}\right)$

SOLUTION:

Let $P(r, \theta) = \left(4, \frac{11\pi}{4}\right)$.

We need to subtract $\frac{11\pi}{4}$ by πk, such that the result is between 0 and π. Test multiples of π.

$$\begin{aligned}\frac{11\pi}{4} - 2\pi &= \frac{33\pi}{12} - \frac{24\pi}{12} \\ &= \frac{9\pi}{12} \\ &= \frac{3\pi}{4}\end{aligned}$$

When k is even, use r for r to obtain the correct polar coordinates. .

$$P\left(4, \frac{11\pi}{4}\right) = \left(4, \frac{3\pi}{4}\right)$$

47. (1.25, –920°)

SOLUTION:

Let $P(r, \theta)$ = (1.25, –920°).

We need to add $180k$ to –920°, such that the result is between 0 and 180. Test multiples of 180.

$$\begin{aligned}-920° + 180(6) &= -920° + 1080 \\ &= 160\end{aligned}$$

When k is even, use r for r to obtain the correct polar coordinates.

P(1.25, –920°) = (1.25, 160°)

48. $\left(-1, -\frac{21\pi}{8}\right)$

SOLUTION:

Let $P(r, \theta) = \left(-1, -\frac{21\pi}{8}\right)$.

We need to add π to $-\frac{21\pi}{8}$, such that the results is between 0 and π. Test multiples of π.

$$\begin{aligned}-\frac{21\pi}{8} + 3\pi &= -\frac{21\pi}{8} + \frac{24\pi}{8} \\ &= \frac{3\pi}{8}\end{aligned}$$

When k is odd , use $-r$ for r to obtain the correct polar coordinates.

$$P\left(-1, -\frac{21\pi}{8}\right) = \left(1, \frac{3\pi}{8}\right)$$

49. (−6, −1460°)

SOLUTION:

Let $P(r, \theta)$ = (−6, −1460°).

We need to add 180k to −1460°, such that the result is between 0 and 180°. Test multiples of 180°.

$$\begin{aligned}-1460° + 9 \cdot 180° &= -1460° + 1620° \\ &= 160°\end{aligned}$$

Since k is odd, we need to replace r with $-r$ to obtain the correct polar coordinates.

$$P(-6, -1460°) = (6, 160°)$$

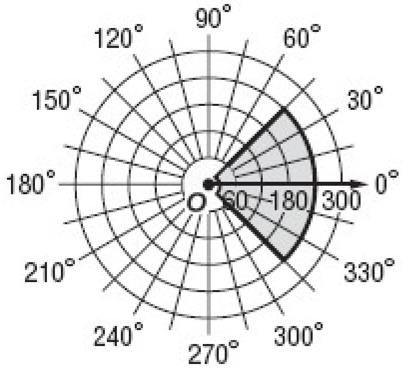
50. **AMPHITHEATER** Suppose a singer is performing at an amphitheater. We can model this situation with polar coordinates by assuming that the singer is standing at the pole and is facing the direction of the polar axis. The seats can then be described as occupying the area defined by $-45° \le \theta \le 45°$ and $30 \le r \le 240$, where r is measured in feet.

a. Sketch a graph of this region on a polar grid.

b. If each person needs 5 square feet of space, how many seats can fit in the amphitheater?

SOLUTION:

a. The parameters $-45° \leq \theta \leq 45°$ and $30 \leq r \leq 240$ describe a portion of a sector of a circle from $-45°$ to 45°. This region is the sector from –45° to 45° with radius 240 units with the sector from –45° to 45° with radius 30 units removed.



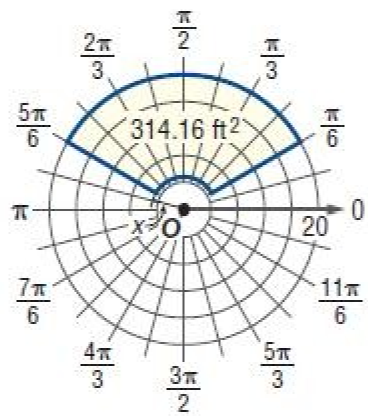
b. The area of a region is the difference of the areas of the two sectors. The area of a sector of a circle with radius r and central angle θ is $A = \frac{1}{2}r^2\theta$, where θ is measured in radians. The angle of each sector is 90°, which is $\frac{\pi}{2}$ radians. Therefore, the area A of the region is given below.

$$\begin{aligned} A_{\text{region}} &= A_{\text{Large Sector}} - A_{\text{Small Sector}} \\ &= \frac{1}{2}(240)^2\left(\frac{\pi}{2}\right) - \frac{1}{2}(30)^2\left(\frac{\pi}{2}\right) \\ &= \frac{\pi}{4}(240^2 - 30^2) \\ &\approx 44{,}532.1 \end{aligned}$$

The area is about 44,532.1 ft^2. Each person needs 5 ft^2 of space.
44,532.1 ft^2 ÷ 5 ft^2/seat ≈ 8906.42 seats

Since it is not possible to have a fraction of a seat, the amphitheater can fit about 8906 seats.

51. **SECURITY** A security light mounted above a house illuminates part of a circular region defined by $\frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6}$ and $x \leq r \leq 20$, where r is measured in feet. If the total area of the region is approximately 314.16 square feet, find x.



SOLUTION:

The area of a sector of a circle with radius r and central angle θ is $A = \frac{1}{2}r^2\theta$, where θ is measured in radians. The area of the region described is the difference of the areas of two sectors. Both sectors have an angle measure of $\frac{5\pi}{6} - \frac{\pi}{6}$ or $\frac{2\pi}{3}$, but the larger has a radius of 20 and the radius of the smaller is x.

$$\begin{aligned} A_{\text{region}} &= A_{\text{Large Sector}} - A_{\text{Small Sector}} \\ &= \frac{1}{2}(20)^2\left(\frac{2\pi}{3}\right) - x^2\left(\frac{2\pi}{3}\right) \\ &= \frac{\pi}{3}(400) - \frac{\pi}{3}x^2 \\ &= \frac{\pi}{3}(400 - x^2) \end{aligned}$$

We are given that the area of the region is approximately 314.6 ft^2. Substitute this value in for A_{region} in the equation above and solve for x.

$$\begin{aligned} A_{\text{region}} &= \frac{\pi}{3}(400 - x^2) \\ 314.6 &= \frac{\pi}{3}(400 - x^2) \\ \frac{943.8}{\pi} &= 400 - x^2 \\ \frac{943.8}{\pi} - 400 &= -x^2 \\ -\frac{943.8}{\pi} + 400 &= x^2 \\ \pm\sqrt{-\frac{943.8}{\pi} + 400} &= x \\ 10 &\approx x \end{aligned}$$

So x is about 10 feet.

Find a value for the missing coordinate that satisfies the following condition.

52. $P_1 = (3, 35°)$; $P_2 = (r, 75°)$; $P_1P_2 = 4.174$

SOLUTION:

Substitute the given information into the Polar Distance Formula.

$$\begin{aligned} P_1P_2 &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)} \\ 4.174 &= \sqrt{3^2 + r^2 - 2(3)(r)\cos(75° - 35°)} \\ 4.174^2 &= 9 + r^2 - 6r\cos 40° \\ 17.422276 - 9 &= r^2 - 6r\cos 40° \\ 8.422276 &= r^2 - 6r\cos 40° \\ 0 &= r^2 - (6\cos 40°)r - 8.422276 \end{aligned}$$

Use the Quadratic Formula to solve for r.

$$\begin{aligned} r &= \frac{6\cos 40° \pm \sqrt{(-6\cos 40°)^2 - 4(1)(-8.422276)}}{2(1)} \\ &\approx -1.404 \text{ or } 6 \end{aligned}$$

53. $P_1 = (5, 125°)$; $P_2 = (2, \theta)$; $P_1P_2 = 4$; $0 \le \theta \le 180°$

SOLUTION:

Substitute the given information into the Polar Distance Formula.

$$\begin{aligned} P_1P_2 &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)} \\ 4 &= \sqrt{5^2 + 2^2 - 2(5)(2)\cos(\theta - 125°)} \\ 4^2 &= 5^2 + 2^2 - 2(5)(2)\cos(\theta - 125°) \\ 16 &= 29 - 20\cos(\theta - 125°) \\ -13 &= -20\cos(\theta - 125°) \\ \frac{13}{20} &= \cos(\theta - 125°) \\ \theta - 125° &= \cos^{-1}\frac{13}{20} \\ \theta &= \cos^{-1}\frac{13}{20} + 125° \\ \theta &\approx 174.46° \end{aligned}$$

54. $P_1 = (3, \theta)$; $P_2 = \left(4, \frac{7\pi}{9}\right)$; $P_1P_2 = 5$; $0 \le \theta \le \pi$

SOLUTION:

Substitute the given information into the Polar Distance Formula.

$$\begin{aligned} P_1P_2 &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)} \\ 5 &= \sqrt{3^2 + 4^2 - 2(3)(4)\cos\left(\frac{7\pi}{9} - \theta\right)} \\ 25 &= 25 - 24\cos\left(\frac{7\pi}{9} - \theta\right) \\ 0 &= -24\cos\left(\frac{7\pi}{9} - \theta\right) \\ 0 &= \cos\left(\frac{7\pi}{9} - \theta\right) \\ \cos^{-1} 0 &= \frac{7\pi}{9} - \theta \\ \frac{\pi}{2} &= \frac{7\pi}{9} - \theta \\ \theta &= \frac{14\pi}{18} - \frac{9\pi}{18} \\ \theta &= \frac{5\pi}{18} \end{aligned}$$

55. $P_1 = (r, 120°)$; $P_2 = (4, 160°)$; $P_1P_2 = 3.297$

SOLUTION:

Substitute the given information into the Polar Distance Formula.

$$\begin{aligned} P_1P_2 &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)} \\ 3.297 &= \sqrt{r^2 + 4^2 - 2(r)(4)\cos(160° - 120°)} \\ 3.297^2 &= r^2 + 16 - 8r\cos 40° \\ &= r^2 - 8\cos 40° r + (16 - 3.297^2) \end{aligned}$$

Use the Quadratic Formula to solve for r.

$$\begin{aligned} r &= \frac{8\cos 40° \pm \sqrt{(-8\cos 40°)^2 - 4(1)(16 - 3.297^2)}}{2(1)} \\ &\approx 1 \text{ or } 5.13 \end{aligned}$$

56. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between polar coordinates and rectangular coordinates.

a. GRAPHICAL Plot points $A\left(2, \frac{\pi}{3}\right)$ and $B\left(4, \frac{5\pi}{6}\right)$ on a polar grid. Sketch a rectangular coordinate system on top of the polar grid so that the origins coincide and the x-axis aligns with the polar axis. The y-axis should align with the line $\theta = \frac{\pi}{2}$.

Form one right triangle by connecting point A to the origin and perpendicular to the x-axis. Form another right triangle by connecting point B to the origin and perpendicular to the x-axis

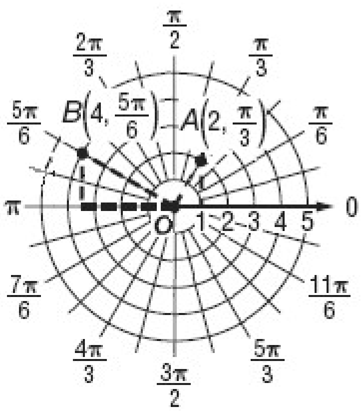
b. NUMERICAL Calculate the lengths of the legs of each triangle.

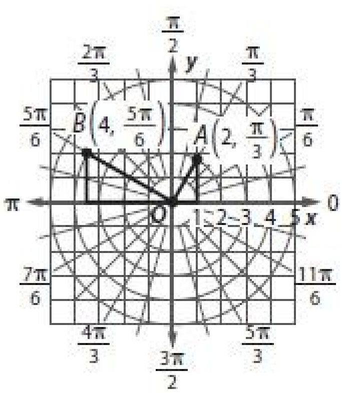
c. ANALYTICAL How do the lengths of the legs relate to rectangular coordinates for each point?

d. ANALYTICAL Explain the relationship between the polar coordinates (r, θ) and the rectangular coordinates (x, y).

SOLUTION:

a.



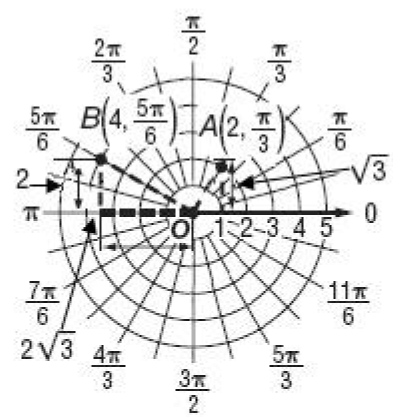


b. For point $A\left(2, \frac{\pi}{3}\right)$, $r = 2$ and $\theta = \frac{\pi}{3}$. The side opposite and side adjacent angle A can be found using the trigonometric ratios.

$$\cos\frac{\pi}{3} = \frac{x}{2} \qquad \sin\frac{\pi}{3} = \frac{y}{2}$$
$$2\cos\frac{\pi}{3} = x \qquad 2\sin\frac{\pi}{3} = y$$
$$2\left(\frac{1}{2}\right) = x \qquad 2\left(\frac{\sqrt{3}}{2}\right) = y$$
$$1 = x \qquad \sqrt{3} = y$$

For point $B\left(4, \frac{5\pi}{6}\right)$, $r = 4$ and $\theta = \frac{5\pi}{6}$. The side opposite and side adjacent angle A can be found using the trigonometric ratios.

$$\cos\frac{5\pi}{6} = \frac{x}{4} \qquad \sin\frac{5\pi}{6} = \frac{y}{4}$$
$$4\cos\frac{5\pi}{6} = x \qquad 4\sin\frac{5\pi}{6} = y$$
$$4\left(-\frac{\sqrt{3}}{2}\right) = x \qquad 4\left(\frac{1}{2}\right) = y$$
$$-2\sqrt{3} = x \qquad 2 = y$$



c. The lengths of the legs represent the x- and y-coordinates for each point.

d. Compare the rectangular and polar coordinates of the points.

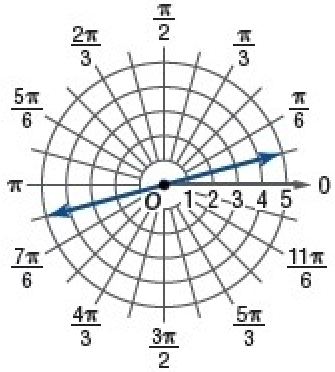
$$A: \left(2, \frac{\pi}{3}\right) = \left(1, \sqrt{3}\right)$$
$$B: \left(4, \frac{5\pi}{6}\right) = \left(-2\sqrt{3}, 2\right)$$

For both points, r corresponds with $r \cos \theta$ and θ corresponds with $r \sin \theta$.

9-1 Polar Coordinates

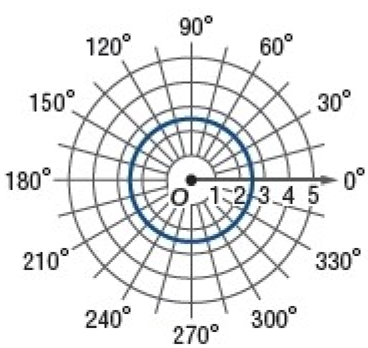
The point with polar coordinates (r, θ) has rectangular coordinates ($r \cos \theta$, $r \sin \theta$).

Write an equation for each polar graph

57. 

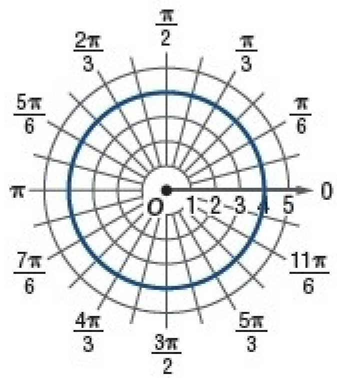
SOLUTION:

The graph consists of all points on the line that make an angle of $\frac{\pi}{12}$ with the positive polar axis, or ordered pairs of the form $\left(r, \frac{\pi}{12}\right)$, where r is any real number. One equation that matches this description is $\theta = \frac{\pi}{12}$.

58. 

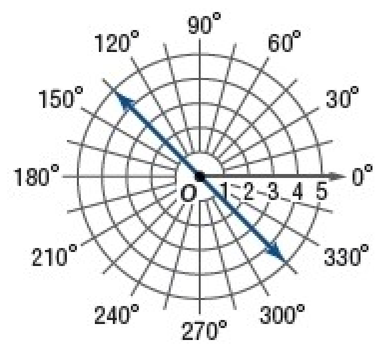
SOLUTION:

The graph is a circle centered at the origin with radius 2.5, or the set of all points that are 2.5 units from the pole. This set of points take on the form $(2.5, \theta)$, where θ is any real angle measure. Two equations match this description: $r = 2.5$ and $r = -2.5$.

59. 

SOLUTION:

The graph is a circle centered at the origin with radius 4, or the set of all points that are 4 units from the pole. This set of points take on the form $(4, \theta)$, where θ is any real number. Two equations match this description: $r = 4$ and $r = -4$.

60. 

SOLUTION:

The graph consists of all points on the line that make an angle 135° with the positive polar axis, or ordered pairs of the form $(r, 135°)$, where r is any real number. One equation that matches this description is $\theta = 135°$.

61. **REASONING** Explain why the order of the points used in the Polar Distance Formula is not important. That is, why can you choose either point to be P_1 and the other to be P_2?

SOLUTION:

Sample answer: The sum and product of the *r*-values are calculated. Both of these operations are commutative. Due to the odd-even identities of trigonometric functions, $\cos(-\theta) = \cos\theta$. Therefore, the order of the angles does not matter.

For example, let two points be (6, 30°) and (4, 60°). Find the distance and interchange the points.

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$$
$$d = \sqrt{6^2 + 3^2 - 2(6)(3)\cos(30 - 60)}$$
$$d = \sqrt{36 + 9 - 36\cos(-30)}$$
$$d = \sqrt{45 - 36\cos(-30)}$$
$$d = \sqrt{45 - 36\left(\frac{\sqrt{3}}{2}\right)}$$
$$d = \sqrt{45 - 18\sqrt{3}}$$

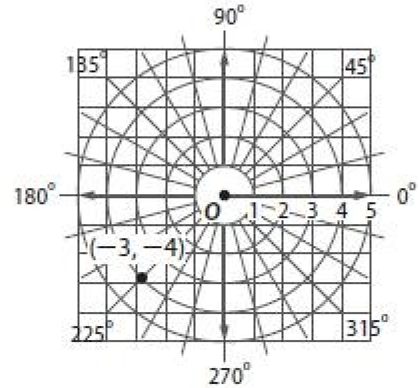
$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$$
$$d = \sqrt{3^2 + 6^2 - 2(3)(6)\cos(60 - 30)}$$
$$d = \sqrt{9 + 36 - 36\cos(30)}$$
$$d = \sqrt{45 - 36\cos(30)}$$
$$d = \sqrt{45 - 36\left(\frac{\sqrt{3}}{2}\right)}$$
$$d = \sqrt{45 - 18\sqrt{3}}$$

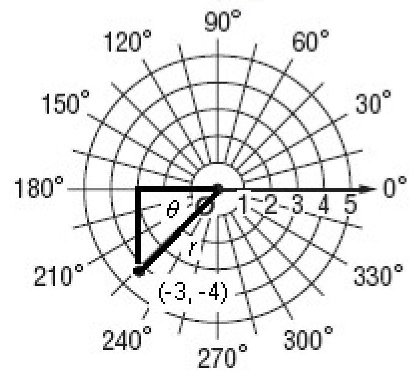
62. **CHALLENGE** Find an ordered pair of polar coordinates to represent the point with rectangular coordinates (−3, −4). Round the angle measure to the nearest degree.

SOLUTION:

Sketch a rectangular coordinate system on top of the polar grid so that the origins coincide and the *x*-axis aligns with the polar axis.

Then plot the point (–3, –4) and connect this point perpendicularly to the *x*-axis and to the origin to form a right triangle as shown.





From the diagram, you can see that the point (–3, –4) lies on the terminal side of a $(180 + \theta)$°-angle with the polar axis as its initial side. To find θ, use the tangent ratio.

$$\tan\theta = \frac{4}{3}$$
$$\theta = \tan^{-1}\frac{4}{3}$$
$$\theta \approx 53°$$

Therefore, point (–3, –4) lies on the terminal side of about a 233°-angle with the polar axis as its initial side.

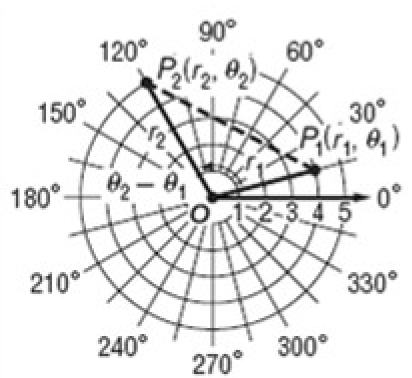
The lengths of the legs of the right triangle shown are 3 and 4, so the hypotenuse *r* of this right triangle is 5.

Therefore, a set of polar coordinates for this point are approximately (5, 233°).

63. **PROOF** Prove that the distance between two points with polar coordinates $P_1(r_1, \theta_1)$ and $P_2(r_2, \theta_2)$ is $P_1P_2 = \sqrt{r_1^{\,2} + r_2^{\,2} - 2r_1r_2\cos(\theta_2 - \theta_1)}$.

SOLUTION:

Sample answer: On a polar grid, graph the points $P_1(r_1, \theta_1)$ and $P_2(r_2, \theta_2)$ with $\theta_2 > \theta_1 > 0$. Connect these points with the origin O to form ΔP_1OP_2. By the definition of polar coordinates, $OP_1 = r_1$ and $OP_2 = r_2$. The measure of angle P_1OP_2 will be given by $\theta_2 - \theta_1$.



Since the measure of two sides and the included angle are known for the triangle, the measure of the third side, P_1P_2, can be found by using the Law of Cosines, $a^2 = b^2 + c^2 - 2bc\cos A$. Therefore by substituting for the measures of the sides and angle,

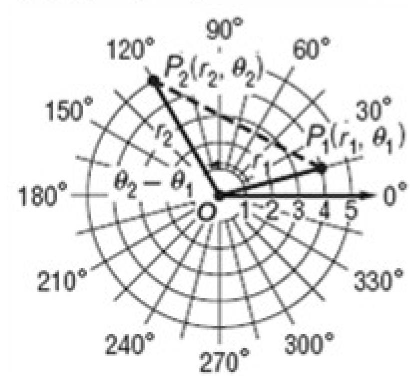
$$(P_1P_2)^2 = r_1^{\,2} + r_2^{\,2} - 2r_1r_2\cos(\theta_2 - \theta_1)$$
$$P_1P_2 = \sqrt{r_1^{\,2} + r_2^{\,2} - 2r_1r_2\cos(\theta_2 - \theta_1)}.$$

64. **REASONING** Describe what happens to the Polar Distance Formula when $\theta_2 - \theta_1 = \frac{\pi}{2}$. Explain this change.

SOLUTION:

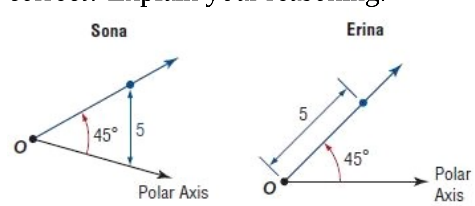
When $\theta_2 - \theta_1 = \frac{\pi}{2}$ the Polar Distance Formula can be simplified.

$$P_1P_2 = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\frac{\pi}{2}}$$
$$P_1P_2 = \sqrt{r_1^2 + r_2^2 - 2r_1r_2(0)}$$
$$P_1P_2 = \sqrt{r_1^2 + r_2^2}$$
$$(P_1P_2)^2 = r_1^2 + r_2^2$$



This relationship indicates that the line segment connecting the P_1 and P_2 is the hypotenuse of the right triangle formed using these two points and the origin as vertices.

65. **ERROR ANALYSIS** Sona and Erina both graphed the polar coordinates (5, 45°). Is either of them correct? Explain your reasoning.



SOLUTION:

Erina; sample answer: Sona plotted a point where the distance between the polar axis and the ray equals 5 units. She should have measured 5 units along the terminal side of the angle.

66. **WRITING IN MATH** Make a conjecture as to why having the polar coordinates for an aircraft is not enough to determine its exact position.

SOLUTION:

Polar coordinates do not take into account the altitude of the aircraft. The angle measure and the ground distance the aircraft is from the radar can be established, but the altitude is needed to determine the aircraft's exact position.

Find the dot product of u and v. Then determine if u and v are orthogonal.

67. **u** = $\langle 4, 10, 1\rangle$, **v** = $\langle -5, 1, 7\rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= \langle 4,10,1\rangle\cdot\langle -5,1,7\rangle\\ &= 4(-5)+10(1)+1(7)\\ &= -20+10+7\\ &= -3\end{aligned}$$

Since $\mathbf{u}\cdot\mathbf{v}\neq 0$, **u** and **v** are not orthogonal.

68. **u** = $\langle -5, 4, 2\rangle$, **v** = $\langle -4, -9, 8\rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= \langle -5,4,2\rangle\cdot\langle -4,-9,8\rangle\\ &= (-5)(-4)+4(-9)+2(8)\\ &= 20-36+16\\ &= 0\end{aligned}$$

Since $\mathbf{u}\cdot\mathbf{v}=0$, **u** and **v** are orthogonal.

69. **u** = $\langle -8, -3, 12\rangle$, **v** = $\langle 4, -6, 0\rangle$

SOLUTION:

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= \langle -8,-3,12\rangle\cdot\langle 4,-6,0\rangle\\ &= (-8)(4)+(-3)(-6)+12(0)\\ &= -32+18+0\\ &= -14\end{aligned}$$

Since $\mathbf{u}\cdot\mathbf{v}\neq 0$, **u** and **v** are not orthogonal.

Find each of the following for a = $\langle -4, 3, -2\rangle$, b = $\langle 2, 5, 1\rangle$, and c = $\langle 3, -6, 5\rangle$.

70. 3**a** + 2**b** + 8**c**

SOLUTION:

$$\begin{aligned}3\mathbf{a}+2\mathbf{b}+8\mathbf{c} &= 3\langle -4,3,-2\rangle+2\langle 2,5,1\rangle+8\langle 3,-6,5\rangle\\ &= \langle -12,9,-6\rangle+\langle 4,10,2\rangle+\langle 24,-48,40\rangle\\ &= \langle 16,-29,36\rangle\end{aligned}$$

71. –2**a** + 4**b** – 5**c**

SOLUTION:

$$\begin{aligned}-2\mathbf{a}+4\mathbf{b}-5\mathbf{c} &= -2\mathbf{a}+4\mathbf{b}+(-5)\mathbf{c}\\ &= (-2)\langle -4,3,-2\rangle+4\langle 2,5,1\rangle+(-5)\langle 3,-6,5\rangle\\ &= \langle 8,-6,4\rangle+\langle 8,20,4\rangle+\langle -15,30,-25\rangle\\ &= \langle 1,44,-17\rangle\end{aligned}$$

72. 5**a** – 9**b** + **c**

SOLUTION:

$$\begin{aligned}5\mathbf{a}-9\mathbf{b}+\mathbf{c} &= 5\mathbf{a}+(-9)\mathbf{b}+1\mathbf{c}\\ &= 5\langle -4,3,-2\rangle+(-9)\langle 2,5,1\rangle+1\langle 3,-6,5\rangle\\ &= \langle -20,15,-10\rangle+\langle -18,-45,-9\rangle+\langle 3,-6,5\rangle\\ &= \langle -35,-36,-14\rangle\end{aligned}$$

For each equation, identify the vertex, focus, axis of symmetry, and directrix. Then graph the parabola.

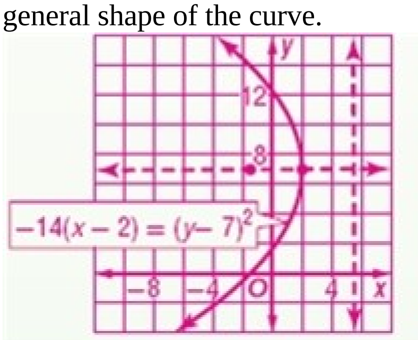
73. $-14(x - 2) = (y - 7)^2$

SOLUTION:

The equation is in standard form and the squared term is y, which means that the parabola opens horizontally. The equation is in the form $(y - k)^2 = 4p(x - h)$, so $h = 2$ and $k = 7$. Because $4p = -14$ and $p = -3.5$, the graph opens to the left. Use the values of h, k, and p to determine the characteristics of the parabola.

vertex: $(h, k) = (2, 7)$
focus: $(h + p, k) = (-1.5, 7)$
directrix: $x = h - p$ or 5.5
axis of symmetry: $y = k$ or 7

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the general shape of the curve.



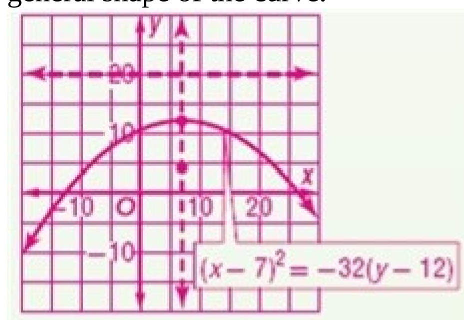
74. $(x - 7)^2 = -32(y - 12)$

SOLUTION:

The equation is in standard form and the squared term is x, which means that the parabola opens vertically. The equation is in the form $(x - h)^2 = 4p(y - k)$, so $h = 7$ and $k = 12$. Because $4p = -32$ and $p = -8$, the graph opens down. Use the values of h, k, and p to determine the characteristics of the parabola.

vertex: $(h, k) = (7, 12)$
focus: $(h, k + p) = (7, 4)$
directrix: $y = k - p$ or 20
axis of symmetry: $x = h$ or 7

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the general shape of the curve.



75. $y = \frac{1}{2}x^2 - 3x + \frac{19}{2}$

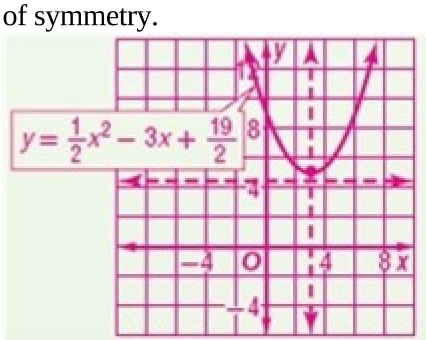
SOLUTION:

$$
\begin{aligned}
y &= \tfrac{1}{2}x^2 - 3x + \tfrac{19}{2} \\
2y &= x^2 - 6x + 19 \\
2y - 19 &= x^2 - 6x \\
2y - 19 + 9 &= x^2 - 6x + 9 \\
2y - 10 &= (x-3)^2 \\
(x-3)^2 &= 2(y-5)
\end{aligned}
$$

The equation is in standard form and the squared term is x, which means that the parabola opens vertically. The equation is in the form $(x - h)^2 = 4p(y - k)$, so $h = 3$ and $k = 5$. Because $4p = 2$ and $p = 0.5$, the graph opens up. Use the values of h, k, and p to determine the characteristics of the parabola.

vertex: $(h, k) = (3, 5)$
focus: $(h, k + p) = (3, 5.5)$
directrix: $y = k - p$ or 4.5
axis of symmetry: $x = h$ or 3

Graph the vertex, focus, axis, and directrix of the parabola. Then make a table of values to graph the curve. The curve should be symmetric about the axis of symmetry.



76. **STATE FAIR** If Curtis and Drew each purchased the number of game and ride tickets shown below, what was the price for each type of ticket?

Person	Ticket Type	Tickets	Total ($)
Curtis	game ride	6 15	93
Drew	game ride	7 12	81

SOLUTION:

Let g = the price of game tickets and let r = the price of ride tickets. Curtis spent a total of $93, so $6g + 15r = 93$. Drew spent $81, so $7g + 12r = 81$.

Find the coefficient matrix A.

$$A = \begin{bmatrix} 6 & 15 \\ 7 & 12 \end{bmatrix}$$

Find the determinant.
$\det(A) = 6(12) - 7(15) = -33$

The determinant does not equal 0 so we can apply Cramer's Rule.

$$
\begin{aligned}
g &= \frac{|A_g|}{|A|} \\
&= \frac{\begin{vmatrix} 93 & 15 \\ 81 & 12 \end{vmatrix}}{-33} \\
&= \frac{93(12) - 81(15)}{-33} \\
&= \frac{-99}{-33} \\
&= 3
\end{aligned}
\qquad
\begin{aligned}
r &= \frac{|A_r|}{|A|} \\
&= \frac{\begin{vmatrix} 6 & 93 \\ 7 & 81 \end{vmatrix}}{-33} \\
&= \frac{6(81) - 7(93)}{-33} \\
&= \frac{-165}{-33} \\
&= 5
\end{aligned}
$$

Therefore, the price of the game tickets was $3 and the price of the ride tickets was $5.

Write the augmented matrix for the system of linear equations.

77. $12w + 14x - 10y = 23$
$4w - 5y + 6z = 33$
$11w - 13x + 2z = -19$
$19x - 6y + 7z = -25$

SOLUTION:

Each of the variables of the system is not represented in each equation. Rewrite the system, aligning the like terms and using 0 for missing terms.

$$\begin{aligned} 12w + 14x - 10y + 0z &= 23 \\ 4w + 0x - 5y + 6z &= 33 \\ 11w - 13x + 0y + 2z &= -8 \\ 0w + 19x - 6y + 7z &= -25 \end{aligned}$$

Write the augmented matrix.

$$\left[\begin{array}{cccc|c} 12 & 14 & -10 & 0 & 23 \\ 4 & 0 & -5 & 6 & 33 \\ 11 & -13 & 0 & 2 & -19 \\ 0 & 19 & -6 & 7 & -25 \end{array}\right]$$

78. $-6x + 2y + 5z = 18$
$5x - 7y + 3z = -8$
$y - 12z = -22$
$8x - 3y + 2z = 9$

SOLUTION:

Each of the variables of the system is not represented in each equation. Rewrite the system, aligning the like terms and using 0 for missing terms.

$$\begin{aligned} -6x + 2y + 5z &= 18 \\ 5x - 7y + 3z &= -8 \\ 0x + y - 12z &= -22 \\ 8x - 3y + 2z &= 9 \end{aligned}$$

Write the augmented matrix.

$$\left[\begin{array}{ccc|c} -6 & 2 & 5 & 18 \\ 5 & -7 & 3 & -8 \\ 0 & 1 & -12 & -22 \\ 8 & -3 & 2 & 9 \end{array}\right]$$

79. $x + 8y - 3z = 25$
$2x - 5y + 11z = 13$
$-5x + 8z = 26$
$y - 4z = 17$

SOLUTION:

Each of the variables of the system is not represented in each equation. Rewrite the system, aligning the like terms and using 0 for missing terms.

$$\begin{aligned} x + 8y - 3z &= 25 \\ 2x - 5y + 11z &= 13 \\ -5x + 0y + 8z &= 26 \\ 0x + y - 4z &= 17 \end{aligned}$$

Write the augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 8 & -3 & 25 \\ 2 & -5 & 11 & 13 \\ -5 & 0 & 8 & 26 \\ 0 & 1 & -4 & 17 \end{array}\right]$$

Solve each equation for all values of *x*.

80. $2\cos^2 x + 5\sin x - 5 = 0$

SOLUTION:

$$\begin{aligned} 2\cos^2 x + 5\sin x - 5 &= 0 \\ 2(1 - \sin^2 x) + 5\sin x - 5 &= 0 \\ 2 - 2\sin^2 x + 5\sin x - 5 &= 0 \\ -2\sin^2 x + 5\sin x - 3 &= 0 \\ 2\sin^2 x - 5\sin x + 3 &= 0 \\ (2\sin x - 3)(\sin x - 1) &= 0 \\ 2\sin x - 3 = 0 \quad &\text{or} \quad \sin x - 1 = 0 \\ \sin x = \frac{3}{2} \qquad & \qquad \sin x = 1 \end{aligned}$$

The equation $\sin x = \frac{3}{2}$ has no real solutions. On the interval $[0, 2\pi)$, $\sin x = 1$ has solution $\frac{\pi}{2}$. Since sine has a period of 2π, the solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{\pi}{2} + 2\pi n,\ n \in \mathbb{Z}$.

9-1 Polar Coordinates

81. $\tan^2 x + 2 \tan x + 1 = 0$

SOLUTION:

$$\begin{aligned} \tan^2 x + 2\tan x + 1 &= 0 \\ (\tan x + 1)^2 &= 0 \\ \tan x + 1 &= 0 \\ \tan x &= -1 \end{aligned}$$

The period of tangent is π, so you only need to find solutions on the interval $[0, \pi)$. The only solution on this interval is $\frac{3\pi}{4}$. Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of π. Therefore, the general form of the solutions is $\frac{3\pi}{4} + \pi n$, $n \in \mathbb{Z}$.

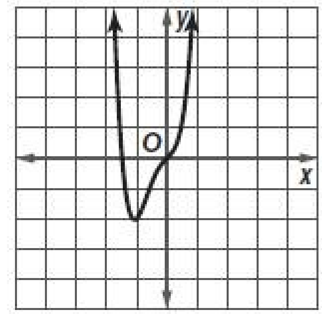
82. $\cos^2 x + 3 \cos x = -2$

SOLUTION:

$$\begin{aligned} \cos^2 x + 3\cos x &= -2 \\ \cos^2 x + 3\cos x + 2 &= 0 \\ (\cos x + 2)(\cos x + 1) &= 0 \\ \cos x + 2 = 0 \quad &\text{or} \quad \cos x + 1 = 0 \\ \cos x = -2 \quad &\quad\;\; \cos x = -1 \end{aligned}$$

The equation $\cos x = -2$ has no real solutions. On the interval $[0, 2\pi)$, $\cos x = -1$ has solution π. Since cosine has a period of 2π, the solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\pi + 2\pi n$ or $(2n + 1)\pi$, $n \in \mathbb{Z}$.

83. **SAT/ACT** If the figure shows part of the graph of $f(x)$, then which of the following could be the range of $f(x)$?



A $\{y \mid y \geq -2\}$
B $\{y \mid y \leq -2\}$
C $\{y \mid -2 < y < 1\}$
D $\{y \mid -2 \leq y \leq 1\}$
E $\{y \mid y > -2\}$

SOLUTION:

The range is all possible y-value of a function. The graph of the function lies on or above the line $y = -2$, therefore, the range of the function is $\{y \mid y \geq -2\}$. The correct answer is A.

84. **REVIEW** Which of the following is the component form of $\overrightarrow{RS}$ with initial point $R(-5, 3)$ and terminal point $S(2, -7)$?

F $\langle 7, -10 \rangle$

G $\langle -3, 10 \rangle$

H $\langle -7, 10 \rangle$

J $\langle -3, -10 \rangle$

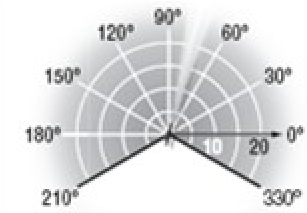
SOLUTION:

$$\begin{aligned} \overrightarrow{RS} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 2 - (-5), -7 - 3 \rangle \\ &= \langle 7, -10 \rangle \end{aligned}$$

The correct answer is F.

9-1 Polar Coordinates

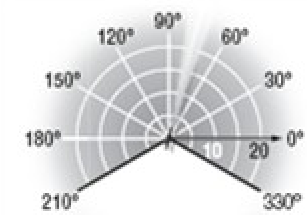
85. The lawn sprinkler shown can cover the part of a circular region determined by the polar inequalities $-30° \le \theta \le 210°$ and $0 \le r \le 20$, where r is measured in feet. What is the approximate area of this region?



A 821 square feet
B 838 square feet
C 852 square feet
D 866 square feet

SOLUTION:

The parameters $-30° \le \theta \le 210°$ and $0 \le r \le 20$ describe a sector of a circle with radius 20 feet from $-30°$ to $210°$.



The area of a sector of a circle with radius r and central angle θ is $A = \frac{1}{2}r^2\theta$, where θ is measured in radians. The angle of the sector is 240°.

$240° \times \frac{\pi}{180°} = \frac{4\pi}{3}$ radians

Therefore, the area of the sector is $\frac{1}{2}(20)^2\left(\frac{4\pi}{3}\right)$ or about 838 ft^2. The correct answer is B.

86. **REVIEW** What type of conic is represented by $25y^2 = 400 + 16x^2$?
F circle
G ellipse
H hyperbola
J parabola

SOLUTION:

Write the equation in standard form.

$$\begin{aligned} 25y^2 &= 400 + 16x^2 \\ -16x^2 + 25y^2 - 400 &= 0 \end{aligned}$$

This equation is of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $A = -16$, $C = 25$, $F = -400$, and $B = D = E = 0$.

Find the value the discriminant, $B^2 - 4AC$.

$$\begin{aligned} B^2 - 4AC &= 0^2 - 4(-16)(25) \\ &= 16{,}000 \end{aligned}$$

Since $B^2 - 4AC > 0$, $25y^2 = 400 + 16x^2$ represents a hyperbola. The correct answer is H.